9950-1206

Report No. 957114-6

JPL
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56P.

FINAL REPORT

UNIFIED CONTROL/STRUCTURE DESIGN AND MODELING RESEARCH

October 31, 1986

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This work was performed for the Jet Propulsion Laboratory, California Institute of Technology, sponsored by the National Aeronautics and Space Administration under contract NAS 7-918.

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CONTENTS

I. Introduction and Overview

Recent years have seen increasing research in integrated control and structural optimization. The primary motivation is control of large flexible space structures, which are becoming larger and more flexible at the same time that their performance requirements are becoming more stringent. The complexity of these structures produces significant uncertainty in the parameters in any model due to changing environments and modeling inaccuracies. Thus control/structure design methods are needed to produce high-performance, robust controllers and light structures.

This report contains the most recent results of work carried out under a JPL sponsored research project dealing with methods for integrated controller design and structural modeling for large space structures. Previous work under this project focused on a method for the simultaneous development of a control law and a design model of a distributed parameter system to be controlled [JPL1, JPL2, JPL3]. The method was based on an LQR approach and it used the notion of functional gains to provide a rationale for generating a finite dimensional design model of appropriate size and modal composition for the control problem at hand. The effectiveness of the method was enhanced by the use of tools such as gain energies and balanced realizations to speed up convergence to the required control law and model [JPL1, JPL2, JPL3]. Related methods were developed for optimal estimator design. Balanced realization theory was subsequently used to reduce the order of the compensator resulting from the procedure.

An important feature of this approach is that the order and modal composition of the model is automatically adjusted as a function of the performance objectives, disturbance environment, sensor locations and actuator locations. This effectively eliminates problems due to modeling errors resulting from modal truncation (i.e., it eliminates spillover problems). However, there is nothing in the approach that explicitly addresses the issue of modeling errors due to poorly known parameter values (e.g., frequencies, damping constants, mode shapes, etc.). In computer experiments where these methods were used to design compensators for a flexible space antenna, it was found that some designs were quite sensitive to parameter errors of this type. Thus, work was begun to identify or develop methods which would overcome the difficulties caused by parameter errors. These efforts have proceeded along two tracks: (1) methods based on structured uncertainty models, and (2) sensitivity optimization.

One approach suggested in the literature for improving robustness is known as Loop Transfer Recovery (LTR) [D1]. This approach seeks to recover the frequency response associated with full state feedback even though the number of measurements available for feedback is smaller than the order of the state vector. This approach is motivated by the fact that if optimal linear quadratic regulator theory (LQR) is used to generate the full state feedback design, then LTR methods guarantee that the gain and phase margins will approach -1/2 to + ∞ gain margin and at least 60° phase margin. In a classical design context, these gain and phase margins are generally considered to be quite satisfactory as regards robustness. This approach was applied to an antenna model and it was found that a) for lightly damped space structures, gain and phase margins are not reliable indicators of robustness, and b) while standard LTR methods did produce the expected gain and phase margins, they did not provide a satisfactory improvement in robustness. There were, however, some benefits from LTR and these had to do with an increase in the low frequency loop gain which the method produced and the consequent improvement in nominal performance and disturbance rejection. Thus, efforts were directed toward retaining the desirable features of LTR and seeking modifications that would enhance robustness with respect to parameter errors. Structured uncertainty descriptions of modeling errors provided a key to developing such an approach.

In a parallel investigation, a nonlinear programming approach was developed that combines methods for robust compensator design with structural optimization methods to develop an algorithm for integrated control/structure design. This algorithm achieves robustness by sensitivity optimization, which means minimizing the sensitivity of closed-loop eigenvalues with respect to uncertainties in plant parameters. The method can be used also to optimize the shape of structural elements to reduce structural weight. An overall control/structure optimal design can be achieved by using both of these features simultaneously. Appropriate constraints are imposed to ensure that high performance is maintained.

The work outlined above led to five technical papers presented at meetings and/or submitted to journals and also formed the basis of the doctoral dissertations of Paul Blelloch and Armen Adamian, UCLA students who were supported by this contract. Because considerable effort was made to ensure that the technical results of this research were clearly presented in these papers and dissertations, they are the best sources for the anyone who wishes to learn about the work. This report, therefore, consists largely of a collection of these refrences together with some discussion which highlights the main results and tries to establish relationships among the various papers.

II. Technical Results

A. Modified Loop Transfer Recovery for LQG Designs (LQG/LTR)

As noted above, loop transfer recovery is a procedure that seeks to recover the open loop frequency response of full state feedback. The method can be used for any full state feedback design, but it is typically used for full state feedback designs based on linear quadratic regulator (LQR) theory, because these designs are known to have desirable features including sizable gain and phase margins. In the standard procedure, one begins by designing a full state feedback regulator which meets the performance requirements. Since the full state is generally not available for feedback, a Kalman-Bucy filter must be used to obtain an estimate of the state using the available measurements. The full state feedback frequency response is recovered by adding a term to the disturbance noise covariance matrix and allowing this term to get large [D1].

The standard approach just outlined was not found to give satisfactory robustness when applied to a model describing a lightly damped flexible antenna. Several ways of changing the the approach were explored with limited success until a modification based on structured uncertainty models [D2] was tried. Structured uncertainty models represent specific modeling errors (frequency errors, damping errors, mode shape errors, etc.) in the form of an auxiliary multivariable feedback loop on the closed loop system. The form of this loop motivates changes in the performance weighting matrix in the LQR problem and the disturbance noise covariance matrix in the K-B filter problem which consist of adding terms reflecting the specific nature of the uncertainty. Making the additional terms large or small permits a controlled tradeoff between performance and robustness. Thus one can arrive at a suitable balance among a) achieving a desired performance goal, b) achieving loop recovery, c) minimizing the effect of structured uncertainties in the plant model. This is done by varying the relative sizes of the weighting matrices corresponding to a), b) and c). These ideas and the results of their application are reported in Appendix II of this report and in [B1,B2]. A journal paper on this work is in preparation. The results show a marked increase in robustness with only a modest loss in performance. The robustness is not achieved by rolling off the loop gain before the first uncertain modes. Several of the uncertain modes are actively controlled. Numerical results focus on uncertain frequencies, but the methods used can easily be extended to other types of parameter errors.

B. Sensitivity Optimization

This approach to robustness is developed in papers [A1-A4] in Appendix I and in Chapters 5-7 of Appendix III of this report (see also [A5]). The idea is to use nonlinear programming to reduce the sensitivity of the closed-loop eigenvalues with respect to modeling errors, while maintaining sufficiently high performance of the closed-loop control system.

Paper [A1] derives formulas for the sensitivities of closed-loop eigenvalues with respect to uncertain plant parameters and presents a numerical example that demonstrates the effect of these sensitivities on robustness in control of a flexible structure. The analysis in [A1] indicates that the first-order sensitivities of the closed-loop eigenvalues approach infinity as a controller and an estimator eigenvalue approach each other and suggests that robustness can be improved by separating controller and estimator eigenvalues. The numerical results for the flexible structure example in [A1, A2] demonstrate the improved robustness achieved by moving the estimator eigenvalues to the left of the controller eigenvalues.

The research has lead to a general guideline for choosing the state weighting matrix and the process noise covariance (the Q matrices) in the LQG problem to improve robustness: After the state weighting for the control problem is chosen according to performance criteria, the Q matrix for the estimator design is chosen to move the estimator eigenvalues for the controlled modes with higher frequencies sufficiently to the left of the closed-loop controller eigenvalues to reduce the closed-loop eigenvalue sensitivities to acceptable levels. Examples of Q matrices that achieve this sensitivity reduction are given in [A1-A5]. In general, the estimator Q for a modal representation of a structure is diagonal and its diagonal elements increase as the corresponding structural frequencies increase.

Paper [A3, A4] in Appendix I and Chapters 6 and 7 of Appendix III discuss optimal eigenvalue sensitivity reduction in conjunction with optimal weight reduction by structural shape optimization. The idea is to combine minimization of closed-loop eigenvalue sensitivity with optimization of structural mass distribution, subject to constraints on eigenvalue location, to produce a robust controller, a light structure and a closed-loop system with fast response. While the measure of robustness used in the design objective is the first-order sensitivity of the closed-loop eigenvalues, the final evaluation of the robustness of the design is based on large variations in the uncertain parameters. The numerical results in [A3, A4, A5] and Appendix III demonstrate the effectiveness of the method for producing both a robust control system and a light structure.

Among references that address integrated control/structure design are [B3, J1, S1]. The papers [B3, J1] optimize combinations of closed-loop eigenvalue location, control gain magnitudes and structural design subject to constraints on the closed-loop eigenvalues. Reference [S1] minimizes a linear combination of structural mass and a quadratic control performance index subject to frequency constraints. Thus the combination of structural design and eigenvalue location in an integrated optimization problem is not new. The main innovation in [A3, A4, A5] is the presence of a direct measure of robustness in the overall objective functional.

C. Comparison of Results from Modified LQG/LTR Methods and Sensitivity Optimization

While the two approaches taken for robust compensator design in Appendices II and III (or [A5] and [B1]) and the corresponding papers are quite different, they lead to certain common conclusions. One important observation says that closed-loop eigenvalues corresponding to rigid-body modes should not lie to the left of closed-loop eigenvalues corresponding to flexible modes. This indicates that it is not a good idea to apply strong torques to the hub without taking proper care to also control the motion of the attached structure. This result is somewhat counter intuitive, since it says that robustness is improved by exerting relatively more effort in controlling the flexible modes.

Another similarity lies in the way the Q matrices should be chosen in LQG compensator design. The structured uncertainty approach in Appendix II modifies the regulator and estimator Q's by adding additional matrices whose diagonal elements are proportional to the corresponding structural frequencies. In a similar way, the sensitivity reduction in Appendix III requires at least that the estimator Q have increasing diagonal elements, although these elements are not necessarily proportional to the frequencies.

Finally, while the sensitivity reduction approach leads to design guidelines that directly involve separation of regulator and estimator eigenvalues, there is no explicit concern for relative regulator and estimator eigenvalue location in the structured uncertainty approach. However, the similarities between the Q matrices used in the two robust compensator design methods suggest an implicit concern for eigenvalue location in the latter approach. Numerical results support this contention. Examination of the relative locations of closed loop regulator and estimator roots for structured uncertainty based designs reveals that a characteristic of the more robust designs is a wide separation of regulator and estimator eigenvalues. Furthermore, the estimator eigenvalues for flexible modes (but not the rigid body mode) are well to the left of the corresponding regulator eigenvalues. (Compare for example Figs. 4.3.16 and 4.4.19 in Appendix II.) Further investigation of these points should produce better understanding of both approaches, and perhaps better guidelines for robust design of LQG compensators.

D. Additional Measurements and Robustness

In carrying out numerical examples for robustness studies, it was noted that full state feedback LQR designs were very robust to parameter variations in the plant model. For this reason, it was felt that adding more measurements might markedly improve the robustness of estimator based designs. While time did not permit extensive study of this point, a few cases were run to explore the hypothesis in a preliminary way.

The antenna model of Appendix II was used to examine this question. It is essential for the reader to have this reference available to understand the data that will be given here. Case 4.4.4b) in Appendix II was selected as a baseline design because its level of robustness is neither very great nor very poor. This design is based on an eight mode model of one quadrant of the antenna. Figure 1 shows nine possible sensor locations which were considered. Sensor 1 is a rotation sensor at the hub. Sensors 2 and 6 are displacement sensors at the rib tips, and 3 and 7 are displacement sensors at the rib centers. Sensors 4, 5, 8 and 9 are similar displacement sensors on the mesh. The baseline case 4.4.4b) uses position sensors at locations 1 and 2. In building the eight mode model, uncontrollable modes were discarded. As a result, it is a property of the model that measurements at locations 6, 7, 8, and 9, are not independent of those at 2, 3, 4, and 5, respectively. Hence, this small investigation only considers adding position and velocity measurements at locations 1 through 5.

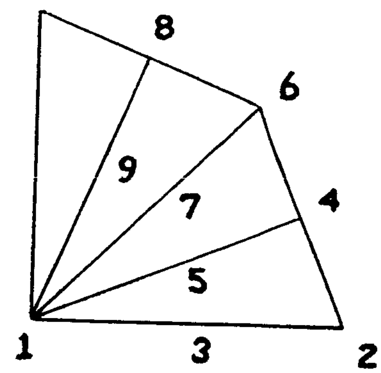

Fig. 1. Antenna Quadrant Model

Table 1 shows how robustness varies when various measurements are used. The notation 1P means a position measurement was used at location 1, while 2V means a velocity measurement was used at location 2, etc. The noise covariances of all sensors was taken to be one, i.e., if the measurement equation is written as

$$y = Cx + n$$

then $E[nn^T]$ is a unit matrix. The phrase "uniform frequency shifts" means all frequencies of the model are shifted together relative to the design model by the indicated percentages. The results show considerable gains in robustness when one begins with one position sensor on the hub (1P) and adds one or two more position sensors on the rib (2P and 3P). However, position measurements on the mesh and velocity measurements almost anywhere do not produce further significant improvement. These results suggest that the benefits of additional measurements is limited, at least within the context of the design procedure that has been developed here. One must keep in mind that even though additional measurements are available, they are still being fed through an estimator based on an erroneous model. This might explain why the robustness of full state feedback is not recovered even when more measurements are added. Much work still needs to be done to better understand the relationship between additional measurements and robustness.

Measurements	Range of Stability (uniform frequency shifts)
1P	-28% to + 5%
1P,2P	-32% to +46%
1P,3P	-27% to +13%
1P,4P	-23% to +13%
1P,5P	-28% to + 5%
1P,2P,3P	-38% to +68%
1P,2P,4P	-32% to +47%
1P,2P,5P	-32% to +46%
1P,1V,2P	-31% to +48%
1P,1V,2P,2V	-31% to +48%

Table 1. Measurements and Robustness

III. Conclusions and Recommendations for Future Research

The problem of designing robust compensators for large flexible space structures is difficult and important. The research summarized here has developed two design methods that have produced robust compensators for the examples to which the methods have been applied, and the analysis underlying the design methods indicates that they should be successful in designing robust compensators for other flexible structures.

Since both of these approaches are new, they are neither fully developed nor understood. As discussed earlier, there appear to be certain connections between the two design methods, but these connections are not clear yet. Further research on the methods should illuminate these connections and reveal improved design methods that combine features of the two approaches developed in this research.

Also, the numerical methods used in both robust design methods need further development and refinement. For the sensitivity optimization method, the nonlinear programming techniques used so far have been generic. More efficient algorithms should be developed that exploit particular characteristics of the sensitivity optimization problem.

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SENSITIVITY OF CLOSED-LOOP EIGENVALUES AND ROBUSTNESS

by

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ABSTRACT

When the compensator for a linear system uses a state estimator, as does an optimal LQG compensator, the relative locations of controller (full state feedback) eigenvalues and estimator eigenvalues can affect robustness significantly. In particular, if an estimator eigenvalue is equal to a controller eigenvalue, then the sensitivity of the closed-loop eigenvalues with respect to uncertain plant parameters is infinite. The sensitivity grows without bound as two such eigenvalues approach one another.

This paper derives the eigenvalue-sensitivity result and presents a numerical example to illustrate the effect of the sensitivity on robustness in control of a flexible structure. The numerical results indicate that avoiding this high sensitivity should be a design criterion in control of flexible structures.

*This research was supported by the Jet Propulsion Laboratory, Pasadena, CA, Grant 957114.

1. Introduction

We have discovered recently that a state-estimator-based compensator for a linear control system produces a closed-loop system whose eigenvalues are very sensitive to parameter errors when any estimator eigenvalues are close to any controller eigenvalues. Indeed, the sensitivity grows without bound as a closed-loop estimator eigenvalue and a closed-loop controller eigenvalue approach each other.

Sections 2 and 3 of this paper show why the high sensitivity arises. An example in section 4 illustrates the effect of the sensitivity on robustness in control of a flexible structure. In this example the parameter errors are in the natural structural frequencies for which the compensator is designed. The numerical results show that the high eigenvalue sensitivity discussed in Section 3 diminishes robustness significantly and that separating controller and estimator eigenvalues improves robustness.

2. The Control System, the Compensator and the Closed-Loop Spectrum

We consider the control system

$$\dot{x} = Ax + Bu \tag{2.1}$$

$$y = Cx \tag{2.2}$$

where the state x(t) is an n-vector, the control u(t) is an m-vector and the measurement y(t) is a p-vector. The $n \times n$ matrix A, the $n \times m$ matrix B and the $p \times n$ matrix C are all real. The compensator is

$$\dot{\hat{x}} = [A - BF - GC]\hat{x} + Gy \tag{2.3}$$

$$u = -F\hat{x} \tag{2.4}$$

where $\hat{x}(t)$ is an estimate of x(t). The gain matrices F and G are determined by some compensator design philosophy. The closed-loop system, shown in Figure 1, satisfies the differential equation

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = A_{c\ell} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}, \tag{2.5}$$

where $A_{c\ell}$ is the $2n \times 2n$ matrix

$$A_{c\ell} = \begin{bmatrix} A & -BF \\ GC & [A - BF - GC] \end{bmatrix}. \tag{2.6}$$

The following standard similarity transformation is useful here:

$$TA_{c\ell}T^{-1} = \begin{bmatrix} [A - BF] & BF \\ 0 & [A - GC] \end{bmatrix} \tag{2.7}$$

where

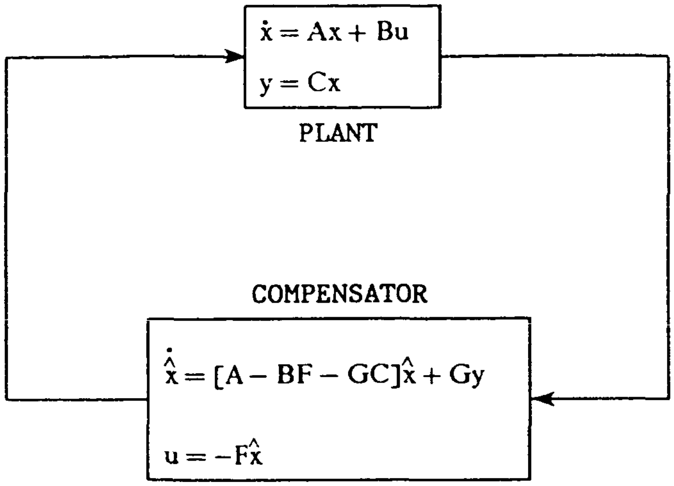

Figure 1. Closed-Loop System.

$$T = T^{-1} = \begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}. \tag{2.8}$$

This transformation shows that, as is well known, the spectrum of $A_{c\ell}$ is the union of the spectrum of $[A - BF]$ and the spectrum of $[A - GC]$. We refer to the eigenvalues of $[A - BF]$ as the controller eigenvalues and to the eigenvalues of $[A - GC]$ as the estimator eigenvalues. Also, from here on, we assume that the eigenvalues of $A_{c\ell}$ are distinct.

Now we derive some formulas involving closed-loop eigenvectors that will be useful in the next section. We denote by X_e the $n \times n$ matrix whose columns are the eigenvectors of $[A - GC]$, by X_c the $n \times n$ matrix whose columns are the eigenvectors of $[A - BF]$, and by Z the $2n \times 2n$ matrix whose columns are the eigenvectors of $A_{c\ell}$. Also, Λ_e is the $n \times n$ diagonal matrix containing the eigenvalues of $[A - GC]$, Λ_c is the $n \times n$ diagonal matrix containing the eigenvalues of $[A - BF]$, and $\Lambda_{c\ell}$ is the $2n \times 2n$ matrix

$$\Lambda_{c\ell} = \begin{bmatrix} \Lambda_c & 0 \\ 0 & \Lambda_e \end{bmatrix}. \tag{2.9}$$

Hence,

$$A_{c\ell} Z = Z \Lambda_{c\ell}, \tag{2.10}$$

and similar equations hold for the estimator and controller eigenvalues and eigenvectors.

It follows from (2.7) and (2.8) that

$$Z = \begin{bmatrix} X_c & X_c\tilde{X} \\ X_c & [X_c\tilde{X} - X_e] \end{bmatrix} \tag{2.11}$$

and

$$Z^{-1} = \begin{bmatrix} [X_c^{-1} - \tilde{X}X_e^{-1}] & \tilde{X}X_e^{-1} \\ X_e^{-1} & -X_e^{-1} \end{bmatrix} \tag{2.12}$$

where the $n \times n$ matrix $\tilde{X}$ satisfies

$$\Lambda_c\tilde{X} - \tilde{X}\Lambda_e = -X_c^{-1}BFX_e. \tag{2.13}$$

There exists a unique solution to (2.13) because, by hypothesis, Λ_c and Λ_e have no eigenvalues in common.

3. Sensitivity of the Closed-Loop Eigenvalues with Respect to Plant Parameters

The preceding section assumes that the plant is known exactly, so that the matrices A, B and C in the compensator are the same as those in the plant. Now we assume that the plant is a function of a parameter β , so that

$$A = A(\beta), \tag{3.1}$$

$$B = B(\beta), \tag{3.2}$$

$$C = C(\beta). \tag{3.3}$$

The compensator is designed for a nominal parameter value β_0, and the closed-loop system is

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = A_{c\ell}(\beta) \begin{bmatrix} x \\ \hat{x} \end{bmatrix}, \tag{3.4}$$

where

$$A_{c\ell}(\beta) = \begin{bmatrix} A(\beta) & -B(\beta)F \\ GC(\beta) & [A(\beta_0) - B(\beta_0)F - GC(\beta_0)] \end{bmatrix}. \tag{3.5}$$

The gains F and G are based on β_0.

When $\beta = \beta_0$, we have the situation in Section 2. Here, we study the first-order sensitivity of the eigenvalues of $A_{c\ell}(\beta)$ with respect to an error between the true plant parameter β and the nominal value β_0 assumed for compensator design. By standard results [L1, P1], we have

$$\Lambda_{c\ell_\beta} = \operatorname{diag}\left[Z^{-1} A_{c\ell_\beta} Z\right], \tag{3.6}$$

where diag [.] means the diagonal matrix with the same diagonal elements, and

$$A_{c\ell_\beta} = \frac{\partial}{\partial\beta} A_{c\ell} = \begin{bmatrix} A_\beta & -B_\beta F \\ GC_\beta & 0 \end{bmatrix}. \tag{3.7}$$

The subscript β always indicates the partial derivative with respect to β. Using (2.11)-(2.12) and carrying out the multiplication in (3.6) yields

$$\Lambda_{c\ell_\beta}(\beta_0) = \frac{\partial}{\partial\beta} \Lambda_{c\ell}(\beta_0) = \text{diag} \begin{bmatrix} \Gamma_1 & 0 \\ 0 & \Gamma_2 \end{bmatrix} \tag{3.8}$$

where

$$\Gamma_1 = X_c^{-1}[A_\beta(\beta_0) - B_\beta(\beta_0)F]X_c - \tilde{X}X_e^{-1}[A_\beta(\beta_0) - B_\beta(\beta_0)F - GC_\beta(\beta_0)]X_c \tag{3.9}$$

and

$$\Gamma_2 = X_e^{-1}B_\beta(\beta_0)FX_e + X_e^{-1}[A_\beta(\beta_0) - B_\beta(\beta_0)F - GC_\beta(\beta_0)]X_c\tilde{X}. \tag{3.10}$$

According to (2.13), the i-j element of the matrix $\tilde{X}$ approaches infinity like the reciprocal of the difference between the i^{th} controller eigenvalue and the j^{th} estimator eigenvalue, except in rare special circumstances. This element of $\tilde{X}$ in general enters the derivative of each closed-loop eigenvalue, according to (3.8), and produces the large sensitivity when estimator eigenvalues are close to controller eigenvalues. Also, when estimator eigenvectors and/or controller eigenvectors are nearly linear dependent, the elements of X_e^{-1} and/or X_c^{-1} approach infinity and produce large sensitivity according to (3.8).

The following section illustrates the effect of eigenvalue-sensitivity on robustness.

4. Example

The structure in Figure 2 consists of a uniform Euler-Bernoulli beam cantilevered to a rigid hub at one end, with a point mass m_1 attached to the other end of the beam. The hub can rotate about its fixed center, point O, and the control is a torque u(t) applied to the hub. There are two sensors, which measure the rigid-body angle θ and the displacement of the point mass m_1, $w(t,\ell)$.

In illustrating the effect on robustness of the eigenvalue sensitivity discussed in Section 3, we use a finite element model of the structure, constructed with three uniform beam elements and cubic B-splines as interpolation functions. (For a given number of degrees of freedom, B-splines approximate the beam more accurately than do Hermite splines. See [G2, R1]). Because cubic B-splines have continuous first and second derivatives, the three-element model of the structure in Figure 2 has four degrees of freedom, including the rigid-body mode.

We model Voigt-Kelvin viscoelastic damping in the beam, which means that the damping matrix is a constant times the stiffness matrix. We take the state vector x(t) to represent the modal displacements and velocities of the three-element/four-mode model, so that the matrix A is

$$A(\beta) = \begin{bmatrix} 0 & I \\ -\beta\Omega^2 & -c_0\Omega^2 \end{bmatrix} \tag{4.1}$$

where Ω is a 4×4 diagonal matrix containing the natural frequencies of the model, c_0 is the damping coefficient and β is an uncertain parameter with nominal value $\beta_0 = 1$. The first element of Ω is zero,

PARAMETER		VALUE	UNIT
hub radius	r	10	in
hub moment of inertia	I_o	10^2	slug.in^2
beam length	ℓ	10^2	in
beam mass per unit length	m_b	10^{-2}	slug/in
2nd moment of cross-sectional area	I	4/3	in^4
modulus of elasticity	E	10^4	slug/in.sec^2
damping coefficient	c_0	10^{-3}	
point mass	m_l	1	slug
undamped fundamental frequency	ω_2	0.967	rad/ sec

Table 1. Structural Data.

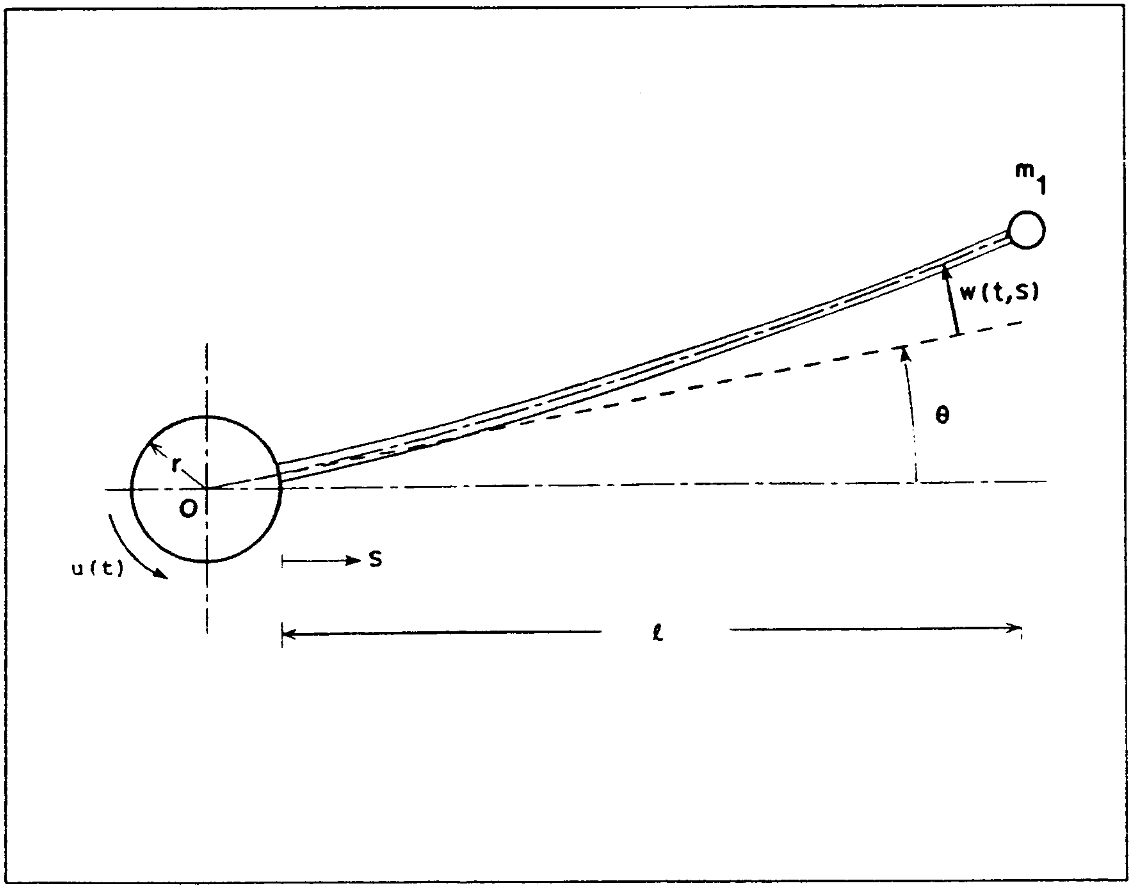

Figure 2. Flexible Structure.

corresponding to the rigid-body mode. When we refer to the natural frequencies of the structure, we will mean the three nonzero elements of Ω only. We assume that the matrices B and C do not depend on β.

Of course, this model may not be sufficiently accurate for designing a compensator for the real structure. In [G1, G2, G3], we have studied the question of how accurate a finite element model is necessary for compensator design and how many modes must be represented in the estimator. While robustness with respect to truncation errors is as important as robustness with respect to parameter errors, we assume here that the three-element model is the structure, to illustrate best the effect on robustness of the eigenvalue sensitivity discussed in the previous section.

For our four-mode model of the structure, we designed a family of linear-quadratic-gaussian (LQG) compensators [K1]. Each compensator has the control gain

$$F = R_c^{-1} B^T P_c \tag{4.2}$$

where the matrix P_c satisfies the Riccati equation

$$P_c[A(\beta_0) + I\alpha_c] + [A(\beta_0) + I\alpha_c]^T P_c - P_c B R_c^{-1} B^T P_c + Q_c = 0 \tag{4.3}$$

The matrix (scalar in this case) R_c penalizes the control in the standard quadratic performance index and the matrix Q_c penalizes the state. The positive scalar α_c guarantees that the eigenvalues of $[A(\beta_0) - BF]$ (the controller eigenvalues) have real parts to the left of $-\alpha_c$. The control gain for all compensators is computed with

$$\alpha_c = 0.2, \tag{4.4}$$

$$R_c = 0.01, \tag{4.5}$$

and Q_c such that

$$x^T Q_c x = 500\theta^2 + 2[\text{Total Energy}]. \tag{4.6}$$

Total energy means kinetic energy plus elastic strain energy in the structure.

The compensators differ in the estimator gains, which are given by

$$G = P_e C^T R_e^{-1} \tag{4.7}$$

where P_e satisfies the Riccati equation

$$[A(\beta_0) + I\alpha_e]P_e + P_e[A(\beta_0) + I\alpha_e]^T - P_e C^T R_e^{-1} C P_e + Q_e = 0. \tag{4.8}$$

Each estimator is a Kalman-Bucy filter for the control system in (2.1)-(2.2) with A replaced by $[A(\beta_0) + I\alpha_e]$ a stationary gaussian process noise with covariance matrix Q_e added to the right side of (2.1) and a stationary gaussian measurement noise with covariance matrix R_e added to the right side of (2.2). The positive scalar α_e guarantees that the eigenvalues of $[A(\beta_0) - GC]$ (the estimator eigenvalues) have real parts to the left of $-\alpha_e$. The estimator gains are computed with

$$\alpha_e = \text{variable} = 0.0,\ 0.2,\ 0.4,\ \ldots,\ 3.8, \tag{4.9}$$

$$R_e = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, \tag{4.10}$$

$$Q_e = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}. \tag{4.11}$$

(Each block in Q_e is a 4×4 matrix).

We designed twenty estimators for the values of α_e indicated in (4.9), and with each of these estimators, we formed the closed-loop matrix $A_{cl}(\beta)$ in (3.5) for a range of β 's. Our measure of robustness for a compensator is how much β can vary, from the nominal value of 1, before the closed-loop system becomes unstable; i.e., before some eigenvalue of $A_{cl}(\beta)$ has nonnegative real part. Figure 3 summarizes the results of the robustness test. The solid line connects the eigenvalues of $[A(\beta_0) - BF]$, which are the same for each compensator. (Only eigenvalues with positive imaginary parts are plotted.) For each compensator, a dashed line connects the eigenvalues of $[A(\beta_0) - GC]$, and the number above each of these estimator eigenvalue plots indicates the percent change in $\sqrt{\beta}$ (from the nominal value of 1) at which the closed-loop system with that compensator becomes unstable. We prefer to look at $\sqrt{\beta}$ because it represents the change in open-loop plant frequencies.

The compensators that place the estimator eigenvalues close to the controller eigenvalues produce a nonrobust closed-loop system, allowing no more than -11% modeling error in the natural frequencies. As the distance between estimator eigenvalues and controller eigenvalues increases, the robustness increases until the compensator will tolerate up to $\pm 22\%$ frequency error and maintain a stable closed-loop system. We have found that the most robust compensator represented in Figure 3 also will tolerate up to $\pm 22\%$ error in any one of the three plant frequencies when the others remain at their nominal values. It is important to note that the robustness increases as the estimator eigenvalues move away from the controller eigenvalues, even though the performance also increases in the sense that estimator errors decay at faster exponential rates.

Eventually, for $\alpha_e > 2.6$, the robustness starts to decrease again. Close examination of our numerical results indicates that the estimator eigenvectors approach linear dependence for the largest values of α_e, so that large terms enter the right sides of (3.9) and (3.10) in the matrix X_e^{-1}. This is another demonstration of the relationship between robustness and sensitivity of closed-loop eigenvalues with respect to parameter errors.

In general, as the real part of a conjugate pair of complex eigenvalues becomes large negatively, the corresponding conjugate pair of eigenvectors become nearly linearly dependent. In our example, this happens first for the eigenvalues nearest the real axis, whose frequency is between 0.035 and 10^{-6} rather than zero, as the graph might suggest. And it happens to a lesser extent for the pair of eigenvalues with frequency approximately 1.

Another reason that the robustness cannot be improved more just by moving all of the estimator eigenvalues farther to the left is that the second-order eigenvalue sensitivities with respect to the uncertain parameter involve the reciprocal of the difference of any two estimator eigenvalues and of any two controller eigenvalues. Because this follows from standard formulas [L1, P1] and is not a result of the special structure of the closed-loop system matrix $A_{c\ell}$, we do not discuss it in detail here. Also, we have found the first-order sensitivities to be more important for robustness. However, the pairs

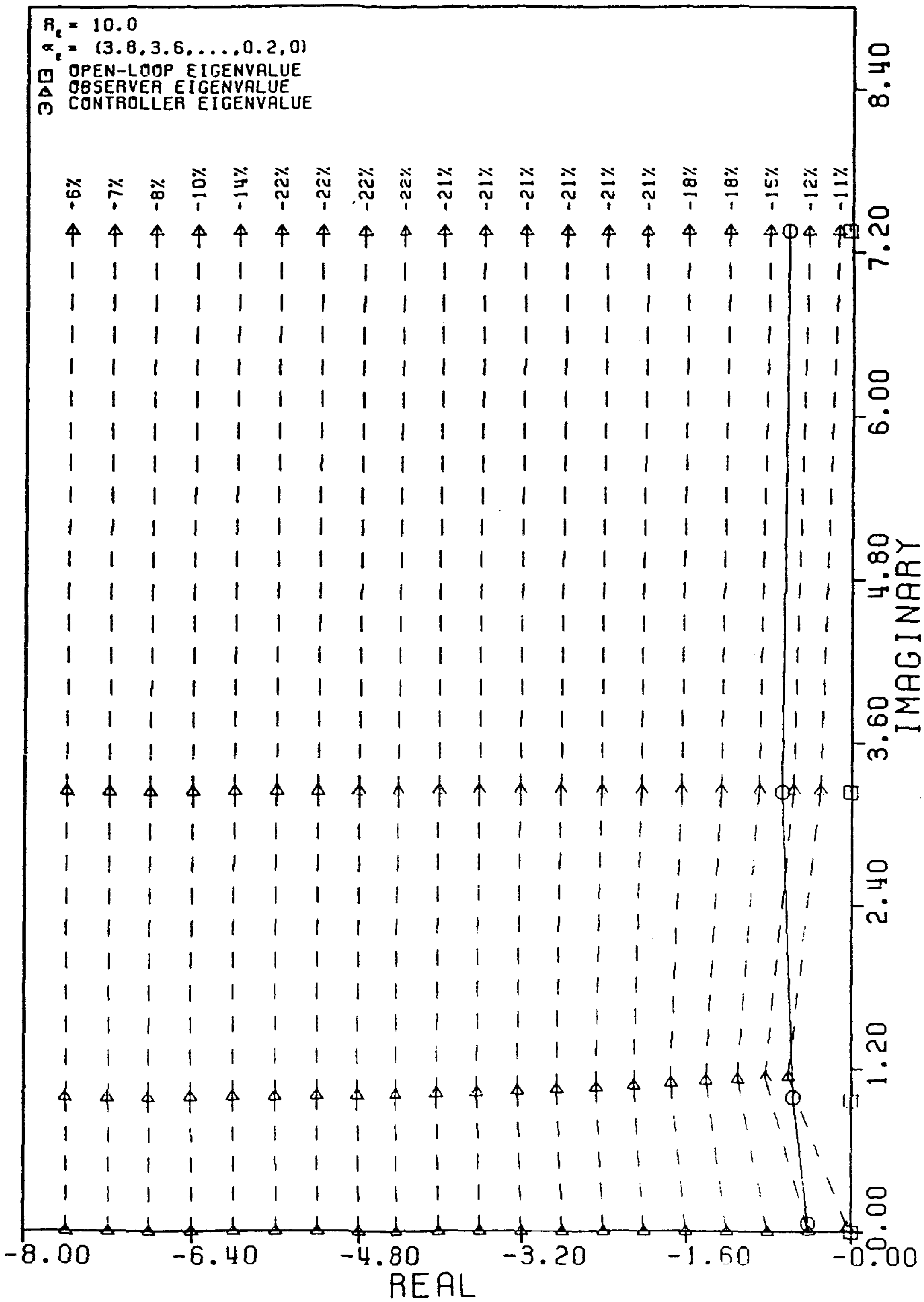

Figure 3. Robustness Test Results.

of controller and estimator eigenvalues near the real axis cause large second-order sensitivity in the closed-loop eigenvalues.

To reduce both the first-order sensitivity produced by almost linearly dependent estimator eigenvectors and the second-order sensitivity produced by closed-loop eigenvalues near the real axis, we designed a new compensator with

$$\alpha_c = 0.2, \tag{4.12}$$

$$R_c = 1.0, \tag{4.13}$$

$$Q_c = \left[\begin{array}{cccc|c} 12 & & & & \\ & 1.25 & & & 0 \\ & & 6.2 & & \\ & & & 35 & \\ \hline & 0 & & & 0 \end{array}\right] \times 1000, \tag{4.14}$$

$$\alpha_e = 0.25, \tag{4.15}$$

$$R_e = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{4.16}$$

$$Q_e = \left[\begin{array}{c|cccc} 0 & & 0 & & \\ \hline & 1 & & & \\ & & 10 & & \\ 0 & & & 10 & \\ & & & & 20 \end{array}\right] \times 100. \tag{4.17}$$

The resulting closed-loop eigenvalues are shown in Table 2. With this compensator, the closed-loop system first becomes unstable at $\sqrt{\beta}$ =

-50%, as opposed to -22% for the most robust compensator represented in Figure 3.

Eigenvalues of $[A(\beta_0) - BF]$	Eigenvalues of $[A(\beta_0) - GC]$
$-0.4221 \pm i0.5805$	$-0.5347 \pm i0.1362$
$-0.5915 \pm i1.0571$	$-1.2888 \pm i2.2618$
$-0.6861 \pm i3.3011$	$-2.2686 \pm i5.7000$
$-0.6773 \pm i7.3835$	$-12.914 \pm i13.902$

Table 2. Closed-Loop Eigenvalues with Robust Compensator.

5. Conclusions

The numerical results for the example illustrate the significant effect that the closed-loop eigenvalue sensitivity derived in Section 3 has on robustness with respect to modeling errors. The results in Section 3 suggest and the example confirms that controller and estimator eigenvalues should be separated for a robust design. Almost linearly dependent estimator eigenvectors or controller eigenvectors diminish robustness also.

In the example, we chose to move the estimator eigenvalues to the left of the controller eigenvalues. While such relative placement of controller and estimator eigenvalues is used frequently in compensator design so that the faster decaying estimator error will make the compensator approximate full-state feedback, we have seen no mention in the literature of the relationship demonstrated here between controller/estimator eigenvalue location and robustness. We have found that, to improve robustness by reducing closed-loop eigenvalue sensitivity, the eigenvalue separation may be achieved as well by placing some or all of the controller eigenvalues sufficiently to the left of nearby estimator eigenvalues or, not surprisingly, by separating imaginary parts of eigenvalues. This is important in controlling complex flexible structures, which often have lightly damped modes along with heavily damped modes, making it impractical to place all estimator eigenvalues to the left of all controller eigenvalues.

Although the analysis in Section 3 and the example in Section 4 deal with a single uncertain parameter, it should be clear that the results apply to any number of parameters. The formulas in Section 3 give the sensitivities of the closed-loop eigenvalues with respect to each parameter. Recently in [A1], we have incorporated the minimization of this sensitivity into the larger problem of integrated control/structure design. The closed-loop eigenvalue sensitivities with respect to all uncertain parameters are included in the overall control/structure objective functional for a numerical optimization problem.

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INTEGRATED CONTROL/STRUCTURE DESIGN AND ROBUSTNESS

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ABSTRACT

When a flexible structure is to be controlled actively, optimum performance is obtained by integrated, or simultaneous, design of the structure and the controller, as opposed to the common practice of designing the structure independently of control consideration and then designing a controller for a fixed structure. The primary design objective from the structural point of view usually is to minimize weight, while the control design objectives depend on the application. An important requirement for a practical control system is robustness with respect to uncertain plant parameters. This paper discusses simultaneous control/structure design when the overall design objective combines the weight of the structure and the robustness of the closed-loop control system. For numerical optimization, robustness is represented by the sensitivity of the closed-loop eigenvalues with respect to uncertain parameters. An example illustrates the optimal design of a flexible structure along with a robust compensator.

RECENT YEARS have seen increasing research in integrated control and structural optimization. The primary motivation of this research is control of large flexible space structures, which are becoming larger and more flexible at the same time that their performance requirements are becoming more stringent. Also, there is a high degree of uncertainty in the parameters of such structures due to changing environments and modeling inaccuracies.

The primary objective of this paper is to design light flexible structure along with robust compensators by addressing the following problem: Find structural parameters in addition to controller and observer gains that minimize an objective function that includes both structural weight and sensitivities of closed-loop eigenvalues with respect to plant uncertainties, subject to eigenvalue constraints. Note that robustness means insensitivity of the closed-loop performance with respect to plant uncertainties. Although there is a vast literature on achieving robust designs using conventional control theory approaches, to our knowledge the proposed approach is new.

The integrated control/structure optimization problem is stated in Section 1 and a numerical example is presented in Section 2.

1. FIRST-ORDER SENSITIVITY AND STRUCTURAL WEIGHT OPTIMIZATION

PROBLEM STATEMENT - Find the elements of h (structural design variables), and the gain matrices F and G (control design variables) that minimize (1.1), which includes the structural weight and the first-order sensitivities of the closed-loop eigenvalues with respect to plant uncertainties (natural frequencies), subject to eigenvalue constraints and partial side constraints on design variables; i.e., choose F, G and h to minimize

$$J(F,G,h) = [J_C(F,G,h)/J_C(F_*,G_*,h_*)] + \alpha[W(h)/W(h_*)] \qquad (1.1)$$

subject to

$$Re(\lambda_C)_i^{\ell} \le \max_i Re(\lambda_{C_i}) \le Re(\lambda_C)_i^{u}$$

$$i = 1, \ldots, 2n, \qquad (1.2)$$

$$Re(\lambda_e)_i^{\ell} \le \max_i Re(\lambda_{e_i}) \le Re(\lambda_e)_i^{u}$$

$$i = 1, \ldots, 2n, \qquad (1.3)$$

$$\min_i |Im(\lambda_C)_i| \ge Im(\lambda_C)_i^{\ell}$$

$$i = 1, \ldots, 2n, \qquad (1.4)$$

$$\min_i |Im(\lambda_e)_i| \ge Im(\lambda_e)_i^{\ell}$$

$$i = 1, \ldots, 2n, \qquad (1.5)$$

$$h_i^{\ell} \leq h_i \leq h_i^{u}$$

$$i = 1, \ldots, n_s, \qquad (1.6)$$

where

$$h = [\, h_1, \ldots, h_{n_s} \,] \qquad (1.7)$$

$$J_c(F,G,h) = \left[\sum_{i=1}^{4n} \gamma_i^2 [2\nabla \mathrm{Re}(\lambda_{c\ell_i})\Omega]^2 \right]^{1/2} \qquad (1.8)$$

$$\nabla = [\partial/\partial\beta_1 \ldots\ldots \partial/\partial\beta_n] \qquad (1.9)$$

$$\Omega^T = [\omega_1^2 \ldots\ldots \omega_n^2] \qquad (1.10)$$

In problems with a rigid-body mode, ω_1, is zero and we use only the sensitivities with respect to the nonzero frequencies in (1.1), so that $\frac{\partial}{\partial\beta_1}$ and ω_1^2 are not included in (1.8) - (1.10).

2. EXAMPLE

Consider the structure shown in Figure 1. An Euler-Bernoulli beam is attached (cantilevered) to a rigid hub at one end and a point mass m_1 is attached to the other end of the beam. The hub can rotate about its fixed center, point O, and the control is a torque u(t) applied to the hub. There are two sensors which measure the rigid body angle θ and the displacement of the point mass m_1, $w(t,\ell)$. The finite element model of this structure was obtained by using three beam elements with variable cross-sectional height and the B-splines as the interpolation functions. (B-splines are piecewise cubic polynomials with continuous second derivatives. See S1). Then the generalized coordinates were transformed to the normal coordinates of the structure and a damping model proportional to the stiffness of the structure was selected. The initial control design was done according to the LQG theory. (See K1). The steady-state optimal control vector u(t) for the LQG problem that minimizes the performance index

$$J = E\{\lim_{T\to\infty} \frac{1}{T} \int_0^T [x^T(t)Q_c x(t) + u^T(t)R_c u(t)]dt\} \qquad (2.1)$$

is generated by the linear control law

$$u(t) = -F\hat{x}(t) \qquad (2.2)$$

where

$$F = R_c^{-1}B^T P \qquad (2.3)$$

is the optimal control gain matrix and the constant nonnegative definite real symmetric matrix P satisfies the algebraic matrix Riccati equation

$$P[A+I\alpha_c]+[A+I\alpha_c]^T P - PBR_c^{-1}B^T P + Q_c = 0 \qquad (2.4)$$

and

$$G = \bar{P}C^T R_e^{-1} \qquad (2.5)$$

is the optimal observer gain matrix, and the constant nonnegative definite real symmetric matrix, $\bar{P}$, covariance matrix of the filtering error, satisfies the alegebraic matrix Riccati equation

$$[A+I\alpha_e]\bar{P} + \bar{P}[A+I\alpha_e]^T - \bar{P}C^T R_e^{-1} C\bar{P} + Q_e = 0 \qquad (2.6)$$

Table 1 shows the initial structural data and Table 2 shows the initial control data, where α_c and α_e are positive scalars which we add to the diagonal elements of the matrix A to move the controller and/or observer eigenvalues to the left of these values.

TABLE 1. Initial Structure Data.

Parameter		Value	Unit
hub radius	r	10	in
hub moment of inertia	I_o	10^2	slug.in^2
beam length	l	10^2	in
beam mass per unit length	m_b	10^{-2}	slug/in
2nd moment of cross-sectional area	I	4/3	in^4
modulus of elasticity	E	$5*10^4$	slug/in.sec^2
proportional damping coefficient	c_o	10^{-3}	
point mass	m	1	slug
fundamental frequency of undamped structure		2.159	rad/sec

Table 2. Initial Control Data

$Q_c(1,1) = 8,000$	$Q_e(5,5) = 10,000$
$Q_c(2,2) = 5,000$	$Q_e(6,6) = 1,000$
$Q_c(3,3) = 10,000$	$Q_e(7,7) = 10,000$
$Q_c(4,4) = 70,000$	$Q_e(8,8) = 40,000$
$R_c = 1.0$	$R_e = I$
$\alpha_c = 0.0$	$\alpha_e = 0.3$

The optimum design was obtained by using the ADS optimizer where the method of feasible directions for constrained minimization and finite difference gradients were selected. Note that the control objective $J_c(F,G,h)$ can be evaluated by doing the numerical analysis in R^{2n} (the space of real 2n-vectors) instead of C^{4n} (the space of complex 4n-vectors). (See A1). Table 3 contains the optimization data used for this example and Table 4 lists the design variables of the initial and the optimized closed-loop designs. Figure 2 contains the iteration history of the control objective $J_c(F,G,h)$ and the structural weight $W(h)$, where J_c is reduced by 70% and W is reduced by 32%.

The robustness of the closed-loop eigenvalues was tested by varying all of the natural frequencies of the plant by a constant percentage while maintaining the original damping of the plant and the original natural frequencies in the compensator. For the initial design, the closed-loop system with the full-state feedback remains stable for ±90% variation in plant frequencies, (full-state feedback means that the entire state vector is measured, so that no estimation is required in the closed-loop system). The closed-loop system with the compensator is unstable for 30% decrease in plant frequencies. For the optimized structure and compensator, the closed-loop system with the full-state feedback and the optimized compensator remain stable for ±90% variation in plant frequencies, which indicates a considerable improvement compared to the robustness of the initial design. Figure 3 shows the closed-loop and the open-loop eigenvalues (with positive imaginary parts) of the initial and the optimized designs, and Figure 4 is the enlarged portion of Figure 3 enclosed by center lines.

Table 3. Optimization Data

$Re(\lambda_c)^u = -0.35$	$Re(\lambda_e)^u = -0.4$
$Re(\lambda_c)^\ell = -120.$	$Re(\lambda_e)^\ell = -120.$
$Im(\lambda_c)^\ell = 0.1$	$Im(\lambda_e)^\ell = 0.1$

$$\gamma_i = \begin{cases} 1 \text{ if } \lambda_{c\ell_i} = \lambda_{c_j} & i = 1, \ldots, 4n \\ \text{and} & j = 1, \ldots, 2n. \\ 1 \text{ if } \lambda_{c\ell_i} = \lambda_{e_j} & i = 1, \ldots, 4n \\ \text{and} & j = 1, \ldots, 2n. \end{cases}$$

$\alpha = 3.7$

$h_i = 1.0 \qquad i = 1, \ldots, n_s$

$h_i^\ell = 0.01 \qquad i = 1, \ldots, n_s$

$h_i^u = 3.0 \qquad i = 1, \ldots, n_s$

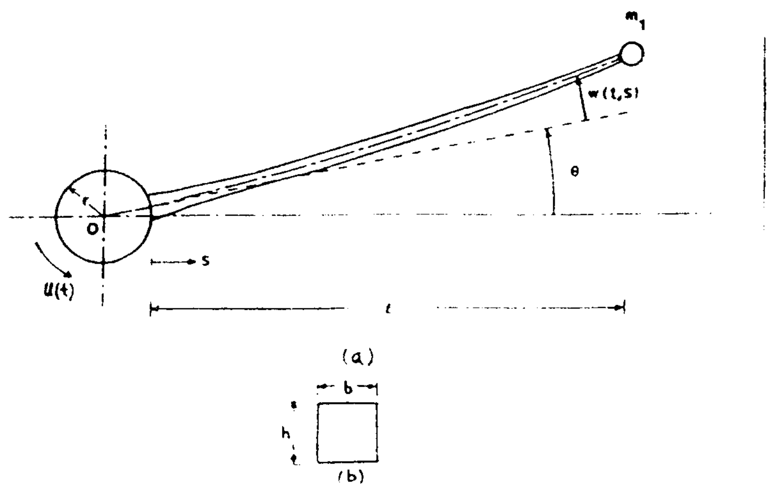

Figure-1 (a) Flexible Structure (b) Beam Cross Section

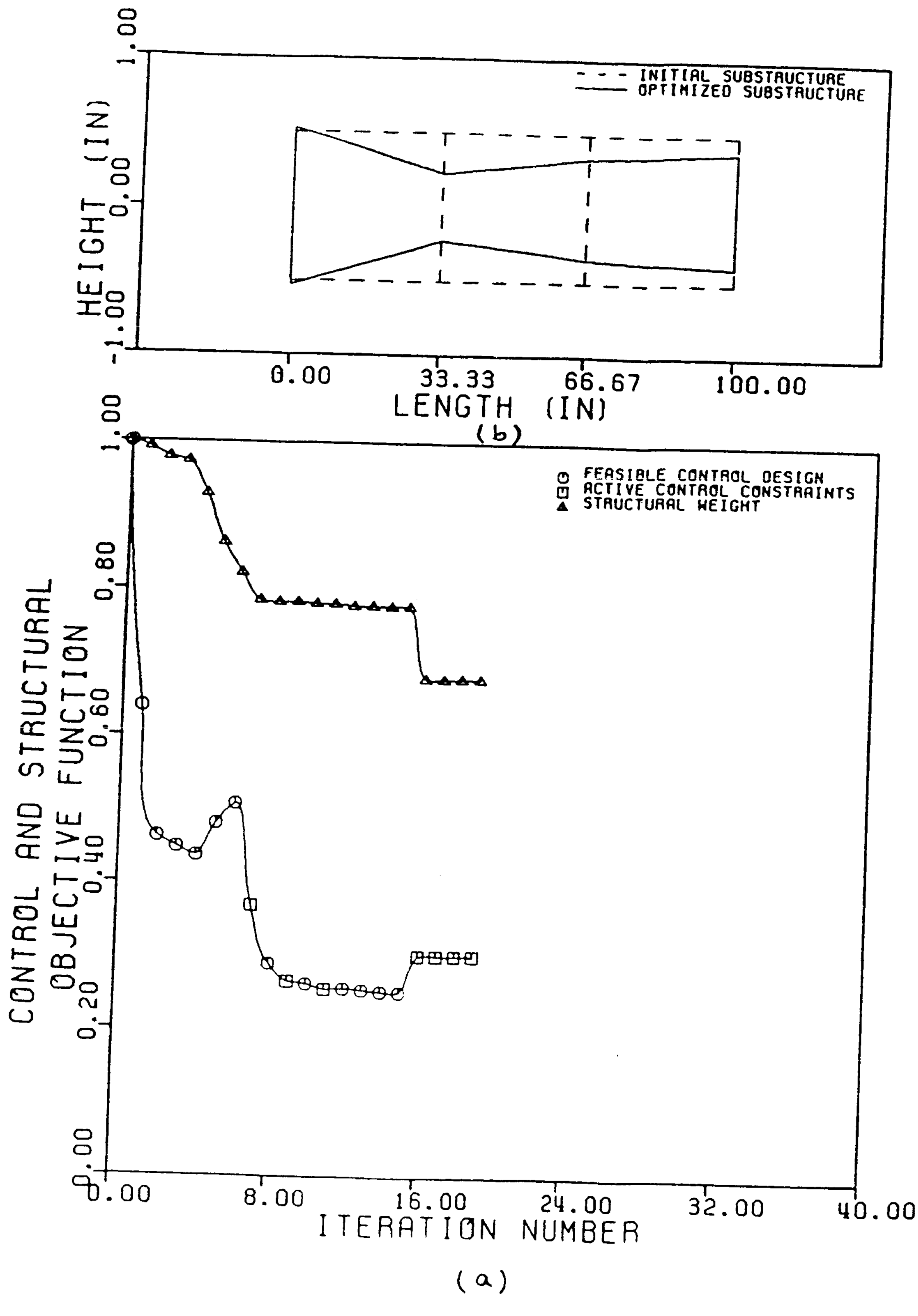

Figure-2.(a)Iteration History of Control and Structural Objective.

(b) Initial and Optimized Beam.

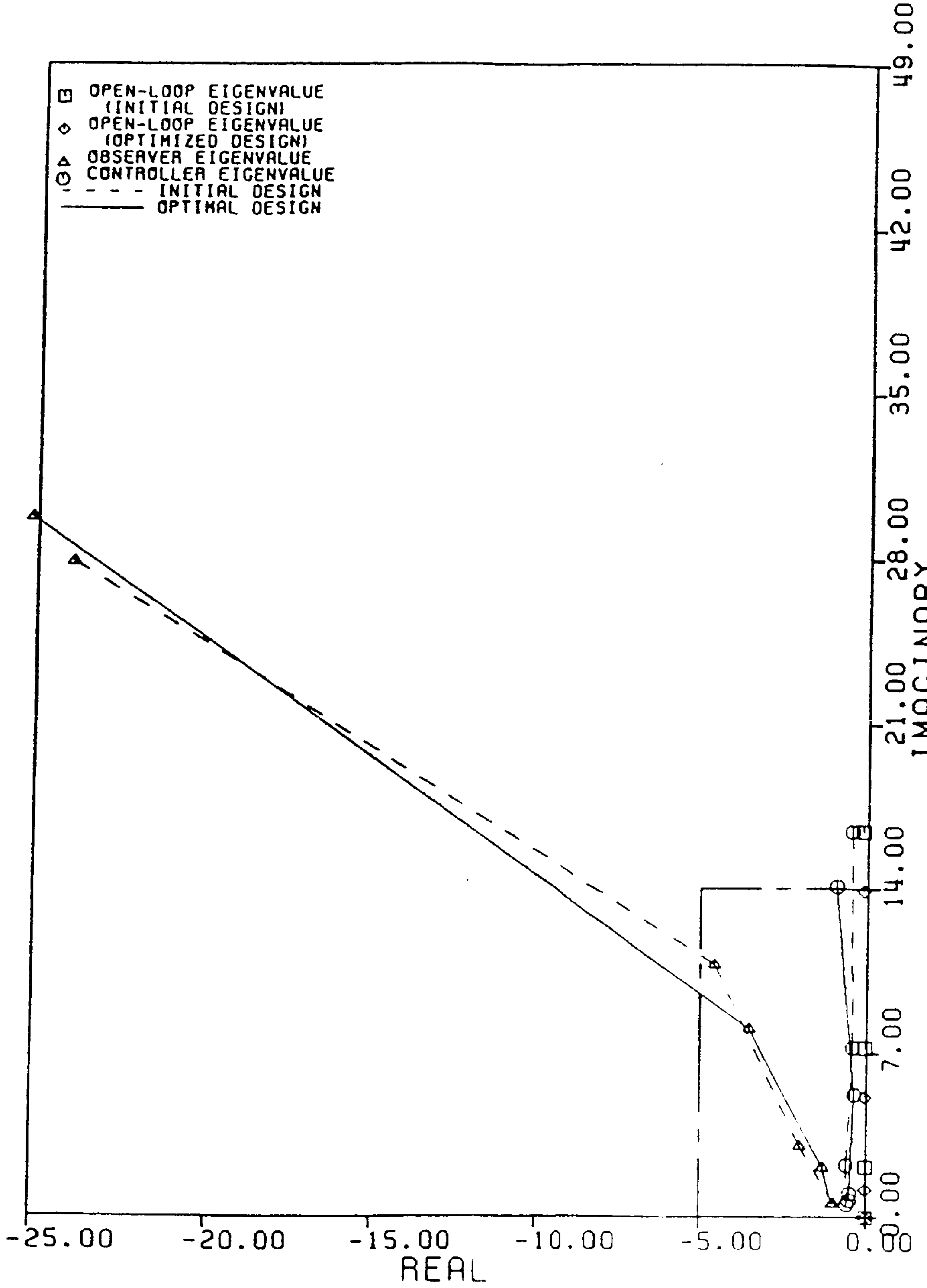

Figure-3. Closed-Loop Eigenvalues of the Initial and the Optimized Design.

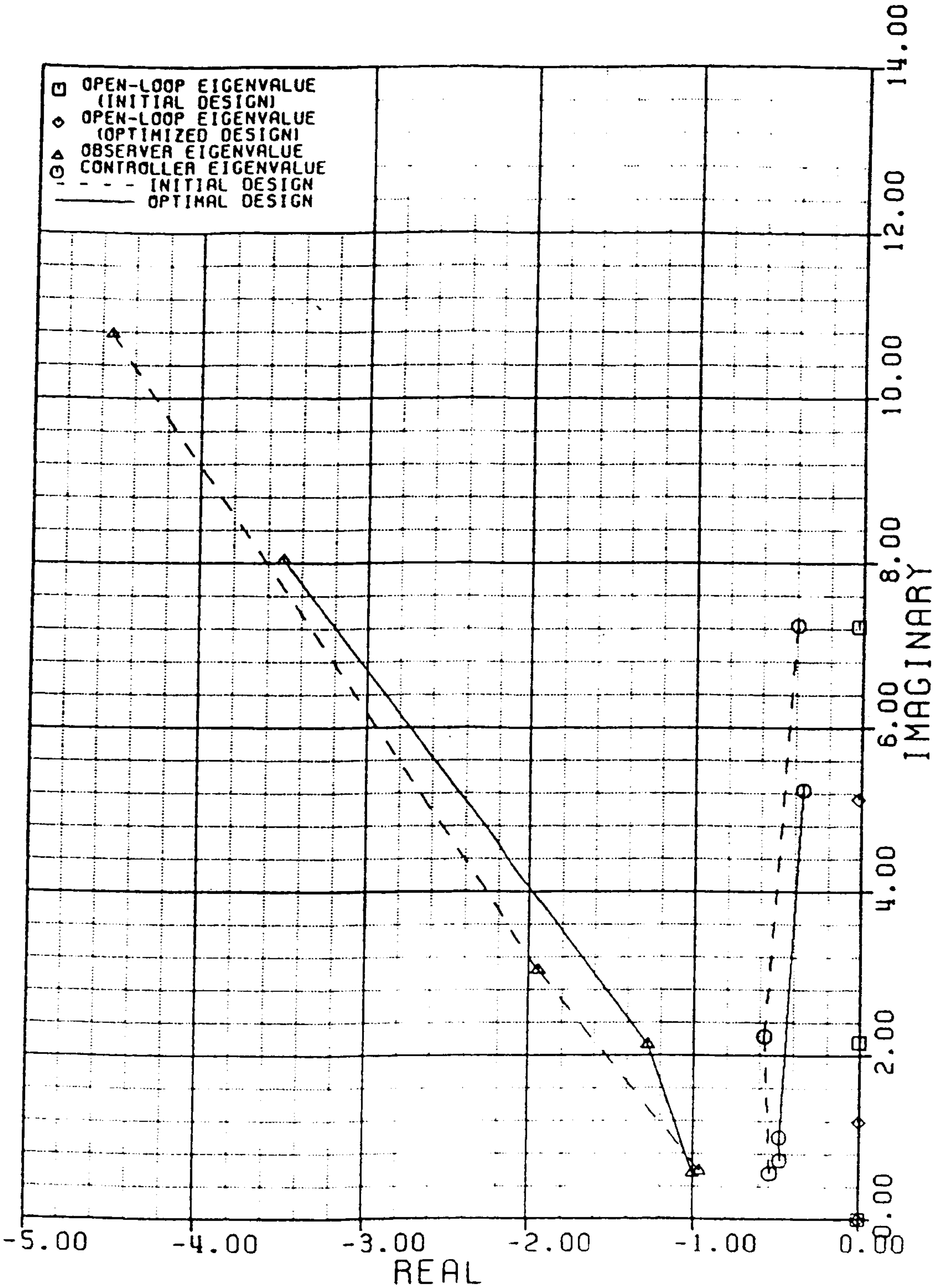

Figure-4. Closed-Loop Eigenvalues of the Initial and the Optimized Design.

Although the objective function of (1.1) and (1.8) assumed that all frequencies vary by the same percentage the closed-loop optimized design remains stable for -70% independent variation in any plant frequency (one at a time). For arbitrary frequency uncertainties, (1.1) and (1.8) can be modified by using the sum of the absolute values of the closed-loop eigenvalue sensitivities with respect to individual frequencies. In addition, the second order sensitivity optimization does not offer considerable improvement compared to the first order one, since the natural frequencies of this example are well separated.

TABLE 4. Design Variables of the Initial and Optimized Closed-Loop Designs.

(a) Cross Sectional Height.

	INITIAL DESIGN	OPTIMIZED DESIGN
j	h_j	h_j
1	1.0	1.065
2	1.0	0.445
3	1.0	0.679
4	1.0	0.767

(b) Controller and Observer Gains

	INITIAL DESIGN			OPTIMIZED DESIGN		
j	F_{1j}	G_{j1}	G_{j2}	F_{1j}	G_{j1}	G_{j2}
1	89.44	247.6	-0.781	88.02	247.6	-0.789
2	-47.42	0.021	15.02	21.01	0.023	14.99
3	-33.13	-0.155	4.302	-51.97	-0.153	4.284
4	-43.35	0.058	-5.449	-152.4	0.060	-5.469
5	180.0	163.3	-0523	183.5	163.4	-0.540
6	-24.11	0.275	20.43	-15.32	27.62	20.43
7	-12.10	-0.967	103.9	-12.01	-0.967	103.9
8	-11.41	-1.882	185.2	-24.84	-1.882	185.2

NOMENCLATURE

n = number of structural modes
m = number of sensors (measurement)
r = number of actuators
n_s = number of structural design variables
$A_{c\ell}$ = 4nx4n closed-loop system matrix
$\lambda_{c\ell}$ = an Eigenvalue of $A_{c\ell}$
A = 2nx2n open-loop system matrix
B = 2nxr actuator influence matrix
C = mx2n measurement matrix
F = rx2n control gain matrix
G = 2nxm observer gain matrix
λ_c = an eigenvalue of (A-BF) matrix (controller eigenvalue)
Q_c = 2nx2n nonnegative definite real symmetric state weighting matrix
R_c = rxr positive definite real symmetric input weighting matrix
α_c = controller alpha shift
λ_e = an eigenvalue of (A-GC) matrix (observer eigenvalue)
Q_e = 2nx2n nonnegative definite real symmetric state excitation noise covariance kernel matrix
R_e = mxm positive definite real symmetric observation noise covariance kernel matrix
α_e = observer alpha shift
$u(t)$ = control vector
ω_i = uncertain plant parameter (natural frequencies)
β_i = ω_i^2
h = structural design variable vector (cross-sectional height)
$J(F,G,h)$ = objective function
$J_c(F,G,h)$ = control objective function
$W(h)$ = structural weight
γ_i = scalar weighting factor
α = scalar weighting factor
E = expected value

ACKNOWLEDGMENT

This research was supported by the Jet Propulsion Laboratory, Pasadena, CA, Grant 957114.

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MODIFIED LTR ROBUST CONTROL FOR FLEXIBLE STRUCTURES

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ABSTRACT

A procedure is developed for dealing with performance and robustness issues in the design of multi-input multi-output compensators for lightly damped flexible structures. The procedure is based upon representing errors in the plant design model as structured uncertainties, and applying a modified version of the Loop Transfer Recovery (LTR) design method. Real parameters errors such as frequency errors, damping errors or modal displacement errors can be treated. The approach involves adjusting the cost function in the regulator problem and the process noise model in the estimator problem in a particular manner which reflects the assumed structure of the modeling errors. Numerical examples dealing with the control of a large flexible space antenna with uncertain frequencies demonstrate a considerable improvement over standard LTR methods. Convenient design parameters can be varied until a satisfactory compromise is achieved between performance and robustness.

I. INTRODUCTION

Robust compensator design for flexible structures involves maintaining closed-loop stability in the face of several types of model errors. Two of the most important are unmodeled, or neglected dynamics and parameter errors in the modeled dynamics. A procedure which addresses the problem of determining the required order of the design model as a function of desired performance is presented in Refs. [1-3]. The present work focuses on the problem of parameter errors in the modeled dynamics. A modified Loop Transfer Recovery (LTR) approach is used to compute a control law which is robust with respect to reasonable plant parameter variations.

In Refs. [1-3], the number of modes required in the design model is determined by the examining the convergence of the compensator as the order of the model is increased. This provides a series of finite-dimensional approximations to the true, infinite-dimensional LQG control law. The convergence of the compensator not only ensures that the finite dimensional compensator will stabilize the infinite-dimensional structure (robustness with respect to unmodeled dynamics), but also indicates that the addition of further modes to the design model will not improve the control design. The sequence in which modes are added depends on their approximate balanced singular values [4,5], a measure of their importance in the input/output map of the system. This method constitutes an infinite-dimensional perspective on the LQG design procedure, while utilizing commonly understood finite-dimensional control design tools.

Once the appropriate reduced order model has been determined, robustness with respect to parameter errors may be addressed within the context of a finite-dimensional problem. Since closed-loop stability seems to depend most strongly on errors in modal frequencies, the present work concentrates on robustness with respect to frequency errors.

Loop Transfer Recovery (LTR) [6] is a design scheme which offers some advantages over other LQG based design approaches. First it recovers the sizable gain and phase margins of full-state feedback LQR designs [7], but more importantly it gives the designer control over loop gain, which implies retention of the desirable performance and disturbance rejection qualities of full-state feedback designs. Although good gain and phase margins are traditionally associated with robustness, examples show that robutness is not necessarily a function of the nominal loop shape. In fact loop shaping is an effective way to achieve robustness only when plant uncertainties are accurately modeled by a single unstructured uncertainty. Lightly damped flexible structures with uncertain modal frequencies on the other hand are systems with highly structured real parameter uncertainties. For systems of this type, achieving robustness by loop shaping alone generally results in an overly conservative design with substantially reduced performance. The modified Loop Transfer Recovery approach presented here maintains some of the advantages of Loop Transfer Recovery but produces a less conservative design which offers a considerable improvement in robustness with respect to parameter errors.

The approach described here was motivated by the μ-synthesis method proposed by Doyle [6,7]. Doyle's method guarantees stability of a closed-loop design for all systems whose dynamics remain within prescribed bounds relative to the nominal design model. The method, however, is substan-

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This work was performed for the Jet Propulsion Laboratory, California Institute of Technology, sponsored by the National Aeronautics and Space Administration under contract NAS 7-918

tially more numerically complex than standard LQG methods. The approach presented here does not guarantee stability for a modeling errors within prescribed bounds, but it does provide a method which permits a controlled trade-off between performance and robustness. Furthermore, it uses standard, well tested numerical methods.

The organization of this paper is a follows. Section II outlines the method for examining compensator convergence as the number of modes in the design model is increased. Section III presents the structured uncertainty representation [8-12] of parameter errors for a flexible structure and outlines the modified Loop Transfer Recovery method. Section IV presents an example based on a wrap-rib antenna [13,14]. Both the (full-state feedback) regulator problem and the estimator problem are presented. Finally we make some conclusions in Section V. The entire procedure is presented in much greater detail in Ref. [15].

II. MODEL REDUCTION

Control design for a large flexible structure must be based on a reduced order model. This model is typically found by truncating "unimportant" modes. One method for selecting the "important" modes of a structure is to examine their approximate balanced singular values [4,5]. These are based on the fact that the modal representation for lightly damped, flexible structures is approximately balanced in the sense of Moore [16]. The singular values take into account frequency, damping and input/output coupling of each mode to give a relative weighting which is intuitively appealing. For a flexible structure with the following modal representation;

$$\ddot{x} + 2Z\Omega\dot{x} + \Omega^2 x = Bu, \quad y=Cx$$
$$Z = \text{diag.}\{\zeta_i\}, \quad \Omega = \text{diag.}\{\omega_i\} \tag{1}$$

the approximate balanced singular values are:

$$\sigma_i = \frac{\sqrt{(b_{i1}{}^2 + \ldots + b_{ip}{}^2)(c_{1i}{}^2 + \ldots + c_{mi}{}^2)}}{4\zeta_i\omega_i} \tag{2}$$

After ordering modes on the basis of approximate balanced singular values, the size of the design model must be chosen. Two different approaches can be taken. One is to ensure that addition of the neglected modes will not destabilize the system. This can be done by treating neglected modes as an unstructured uncertainty and applying the appropriate constraints on loop shape [6]. This is the approach taken in Ref. [17]. However it does not take into account the fact that the model errors in this case are known neglected dynamics rather than uncertainties, and furthermore does not address the issue of performance. The approach taken in this work is to continue adding modes until little further change in the compensator design is observed. One measure for this is to examine the functional gains [1-3]. This method treats the convergence of the LQR and KBF problems separately. A variation on this method which fits well into the context of loop transfer recovery is to examine the loop gain as the model order is increased. This gives a measure on the performance of the overall compensator, and provides a simple but accurate test of the relationship between desired performance and required model order. This involves designing a compensator based on a low order plant, applying this to the "full-order" evaluation model and plotting the loop gain. This procedure is repeated for succesively higher order design models until the loop gain converges. This indicates that the addition of further modes will have no affect on the resulting compensator design.

III. MODIFIED LTR DESIGN

Once an appropriate reduced order model is chosen a design which both provides adequate performance and is robustly stable for all sets of possible plant variations must be chosen. Since closed-loop stability appears to be most sensitive to frequency variations we will assume that this affect dominates any other uncertainties. The method, however, can be trivially extended to include other parameter uncertainties such as damping ratios or mode shapes.

Consider the model of Eq. (1). To place this in the context of the LQG/LTR design approach append a noise model and a quadratic cost functional to arrive at the following control problem:

Given

$$\ddot{x} + 2Z\Omega\dot{x} + \Omega^2 x = Bu + qBw, \quad y=Cx$$

Minimize (3)

$$J = E[\int_0^\infty (x^T L^T L x + u^T u)dt], \quad E[ww^T]=E[vv^T]=I$$

The process noise is assumed to enter at the control inputs in order to achieve loop recovery as $q\to\infty$ [3]. The noise covariances are set to the identity and $u^T u$ is weighted in the cost functional to simplify the problem. While the LQG/LTR approach guarantees excellent gain and phase margins it does not directly take into account any information on parameter uncertainty. In fact, controllers designed via the standard LQG/LTR approach can be extremely sensitive to small variations in the modal frequencies of a lightly damped, flexible structure, as illustrated in Section IV.

One method that formally takes into account model uncertainty is the structured uncertainty representation [8-10]. For models like the one considered here, uncertainty may be represented by a linear fractional transformation on the nominal plant as illustrated in Fig. 1. For m parameter uncertainties the matrix Δ will have the following structure

$$\Delta = \begin{bmatrix} c_1 I_{k_1} & 0 & \cdots & 0 \\ 0 & c_2 I_{k_2} & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & c_m I_{k_m} \end{bmatrix} \tag{4}$$

where the c_i's are real numbers between -1 and 1, and k_i is the rank of the ith uncertainty [11,12]. A method for deriving the interconnection structure is also presented in

Refs. [11,12]. In particular, for the model of Eq. (3) with δ% uncertainty in the diagonal elements of Λ^2, a state-space representation of P_{11} is as follows:

$$\begin{Bmatrix} \dot{x} \\ y \end{Bmatrix} = \left[\begin{array}{cc|c} 0 & I & 0 \\ -\Omega^2 & -2Z\Omega & \Omega \\ \hline \delta\Omega & 0 & 0 \end{array}\right] \begin{Bmatrix} x \\ u \end{Bmatrix} \tag{5}$$

This suggests that one way to deal with frequency uncertainty is to set up the following modified LQG/LTR problem.

Given

$$\ddot{x} + 2Z\Omega\dot{x} + \Omega^2 x = Bu + qBw_1 + r\Omega w_2, \quad y=Cx$$

$$E[w_1 w_1^T] = E[w_2 w_2^T] = E[vv^T] = I$$

Minimize (6)

$$J = E[\int_0^\infty (x^T[qc1L^TL+qc2Q_r]x + u^Tu)dt]$$

where $Q_r = \begin{bmatrix} \Omega^2 & 0 \\ 0 & 0 \end{bmatrix}$

For a given matrix L in Eq. (6) four free parameters remain in the problem. These are qc1 and qc2 in the full-state feedback regulator problem and q and r in the optimal estimator problem. Increasing qc1 increases the emphasis placed on performance, while increasing qc2 increases the emphasis on the uncertainty model. Increasing q improves loop recovery, while increasing r increases the emphasis on the uncertainty model again. As illustrated in the next section, the designer can attain a controlled trade-off between robustness and performance by manipulating these four scalar variables.

IV. EXAMPLE

Consider the large flexible space antenna described in Refs. [13-15], and illustrated in Fig. 2. This is a wrap-rib design of approximately 180ft in diameter. The first fundamental flexible frequency is at 6.95 rad/sec with a damping ratio of 1.1%. The control problem consists of actively damping the out-of-plane motion due to slew maneuvers so as to minimize the overall antenna RMS surface error with respect to its nominal position in space. Actuators provide orthogonal torques at the hub center while measurements of the hub rotation and tip deflections are available. Due to symmetry, the problem can be reduced to the control of a single quadrant with one actuator and two measurements. This is illustrated in Fig. 3.

Using the methods described in Section II an 8-mode design model is chosen. This consists of one rigid body mode and 7 flexible modes. For simplicity the uncertainty in modal frequencies is considered to be equal for all modes.

Now consider the following hypothetical specifications on the input loop shape.

Bandwidth	~5 rad/sec
Loop gain	60db at .1 rad/sec
Phase margin	60°
Gain margin	20db

These might be derived from disturbance rejection considerations (where disturbances act at the actuator inputs), or they might be derived from actuator uncertainty considerations. They can be met by a standard LQG/LTR approach. In particular, assume a control problem in the form of Eq. (6); let r=qc2=0 (ignore the uncertainty model), let qc1=10 and q=10^9. The resulting loop shape is shown in Fig. 4. The gain margin is 36db and the phase margin is 65°. The design minimizes RMS error while also meeting frequency domain performance specifications. However, a simple check shows that a uniform increase of 7% in the modal frequencies results in instability of the closed-loop system. Next take the uncertainty model into account to improve robustness. As a first step add white noise at the uncertainty model input. In this case all parameters remain as before except r which becomes 10^4. The resulting loop shape is identical to that illustrated in Fig. 4, but the closed-loop system is now stable for 35% uniform increases in all modal frequencies. It is also stable of 17% decreases and first goes unstable for a 17% increase in the second modal frequency coupled with 17% decreases in the other six. Robustness can be improved even further by penalizing the output of the uncertainty model. Fix all other parameters and let qc2=1,000. The loop shape for this case is illustrated in Fig. 5. Gain margin is 20db and phase margin is 85°. All frequency domain specifications are still met, but the closed-loop system is now stable for 46% uniform increases in frequency along with 32% decreases. The system first goes unstable for a 32% increase in the fourth modal frequency coupled with 32% decreases in the other six.

The differences between the sensitive and robust designs can be examined from a number of different points of view. The cost functional for the robust design places considerably greater emphasis on the higher modes. This is because the matrix Q_r weights each mode in proportion to its frequency. The matrix L^TL on the other hand places relatively higher weighting on the rigid body mode. The robust design therefore results in closed-loop regulator poles which lie further to the left with increasing frequency. The sensitive design, on the other hand, attempts to push the closed-loop poles corresponding to the rigid body mode further to the left than those corresponding to the flexible modes. This is also demonstrated by the loop gains (Figs. 3 and 4). While both designs have approximately equal low frequency gain, the sensitive design rolls off much more quickly. The robust design has a higher loop gain in the region of uncertain frequencies. These results suggest that designs which attempt a high degree of control of the rigid body mode relative to flexible modes will be very sensitive to frequency uncertainty.

V. CONCLUSIONS

Standard Loop Transfer Recovery methods are an effective way to achieve robust controller designs when the modeling errors of the plant are well characterized by a single unstructured uncertainty model. However, in the case of a flexible structure with uncertain frequencies the unstructured uncertainty model is overly conservative. In this case a modification of the Loop Transfer Recovery procedure is needed. The approach taken here overcomes some of the shortcomings of the standard LTR methods. It sets up the control design procedure in terms of structured uncertainties, and then minimizes the 2-norm of the resulting transfer function. Once measurement noise is added and performance and control cost penalties are appropriately adjusted, a well posed LQG problem is obtained which can be solved with standard numerical methods. Here solving the LQG problem is a computationally efficient approximation to the μ-synthesis approach proposed by Doyle [18,19]. The results of Section IV demonstrate that it does provide a significant improvement over standard LQG/LTR methods. As indicated in Section IV, robustness and performance can be easily traded off by adjusting only four parameters until a suitable compromise is found.

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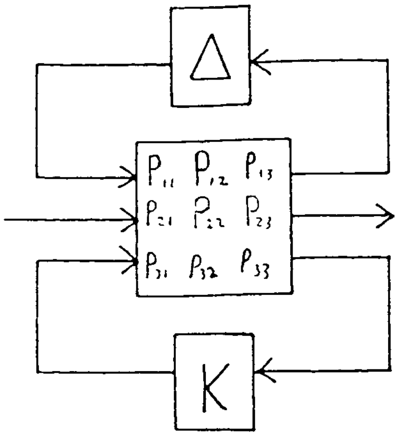
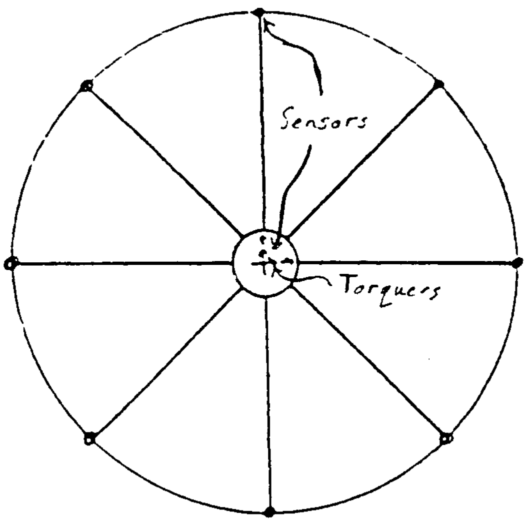


Fig. 1 Structured Uncertainty Representation



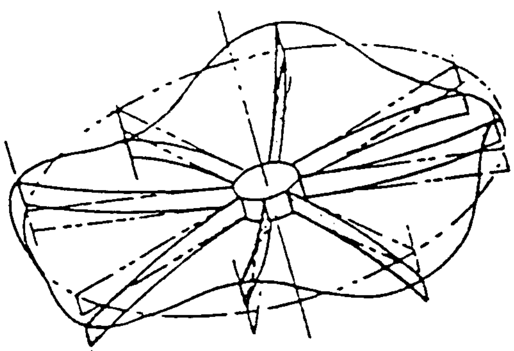


Fig. 2 Full Antenna Model

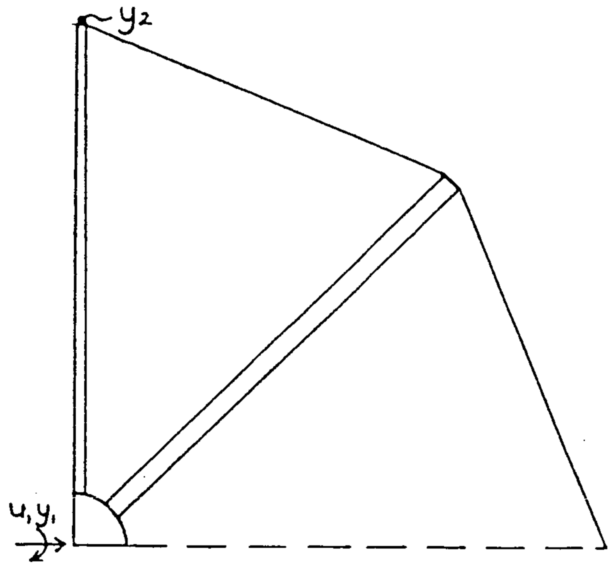


Fig. 3 Quadrant Antenna Model

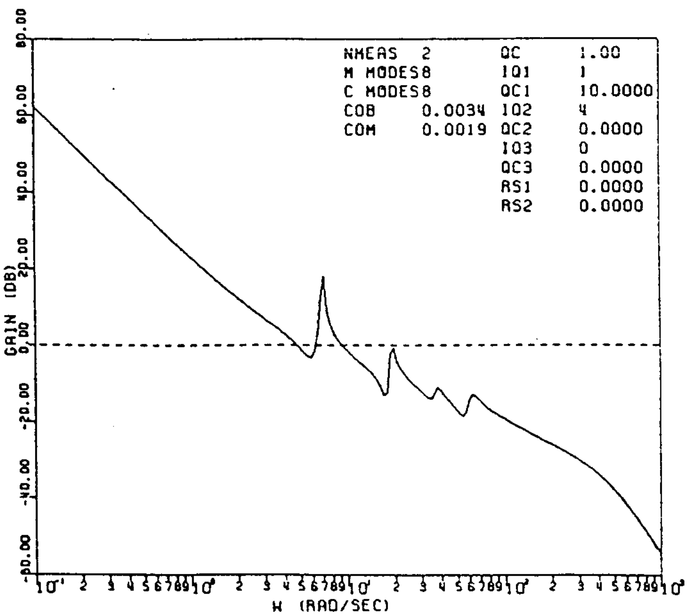


Fig. 4 Loop Shape for Standard LQG/LTR

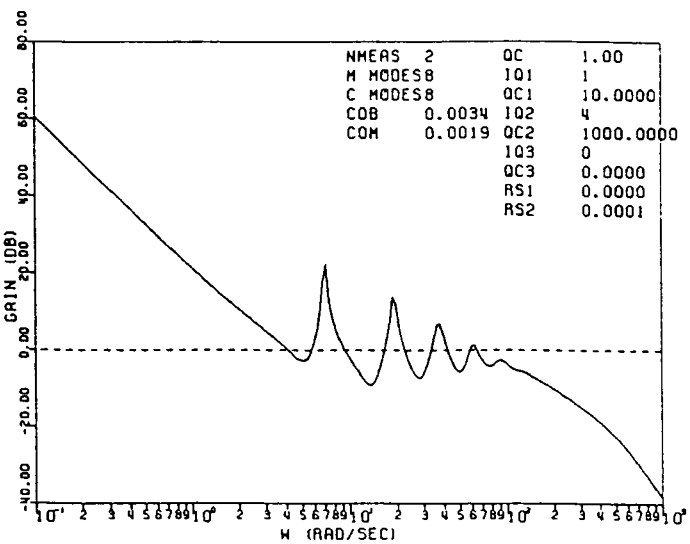


Fig. 5 Loop Shape for Modified LTR